MODULE 11: Probability

Do revision of module 13 in the grade 11 X-Factor.

A. Singel Events

Probability (P): the chance that an event will occur
Sample Space (S): set of all possible outcomes

n**(S): number** of events in the sample space.

Probability of a single event E:

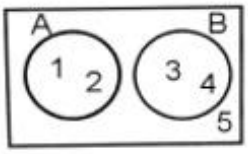
$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

B. More than one event

(A ∩ B) – intersection or (A and B): outcomes which occur in both A and B.

(A ∪ B) – union or (A or B): outcomes which occur in A or B.

1. Mutually exclusive

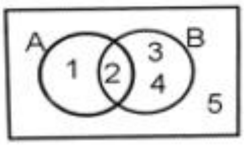


Events cannot take place simultaneously.

P(A and B) = 0

P(A or B) = P(A) +P(B)

2. Not mutually exclusive (inclusive)

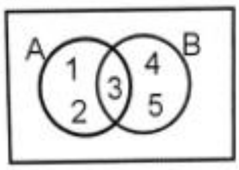


Events can take place simultaneously.

P(A and B) ≠ 0

P(A or B) = P(A) + P(B) – P(A ∩ B)

3. Exhaustive events

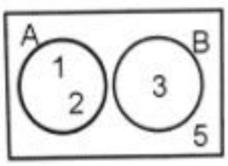


A and B contain all the elements of the sample space.

P(A ∪ B) = 1

Exhaustive events can have an intersection.

4. Complementary events



A' – complement of A.

A and A' are mutually exclusive AND exhaustive.

P(A') = 1 – P(A)

A and B mutually exclusive but not complementary.

Complement of A also includes the number 5. **A' = {3;5}**

C. <u>Independent events (Product-rule)</u>

The outcome of one event will **NOT** have an influence on the outcome of another event.

P(A and B) = P(A) x P(B)

D. <u>Not- independent events</u>

The outcome of one event will have an effect on the outcome of the other event.

P(A and B) ≠ P(A) x P(B)

P(A and B) = P(A) x P(B/A)   [B/A ....B follows A]

(e.g. taking candy from a bag without replacing it.)

Do C and D by using tree diagrams.

E. <u>The Fundamental Counting Principal</u>

If one event can happen in ***m*** ways and another event can happen in ***n*** ways, then both events can happen in ***m* x *n*** ways.

(when coin and dice are cast : 2 x 6 = 12 outcomes)

Examples

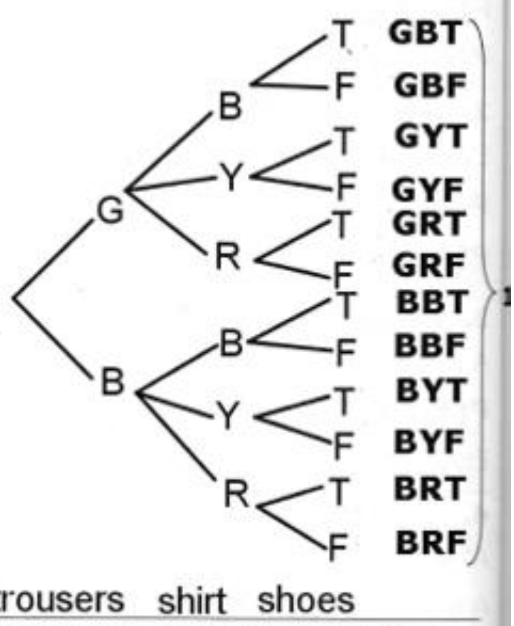
Question

How many different outfits are possible with the following combination of clothes?

Trousers	Shirts	Shoes
Grey Black	Blue yellow Red	Tekkies Flip-flops

Answer

- **With a tree diagram:**



- **With fundamental counting Rule:**

2 x 3 x 2 = 12 options

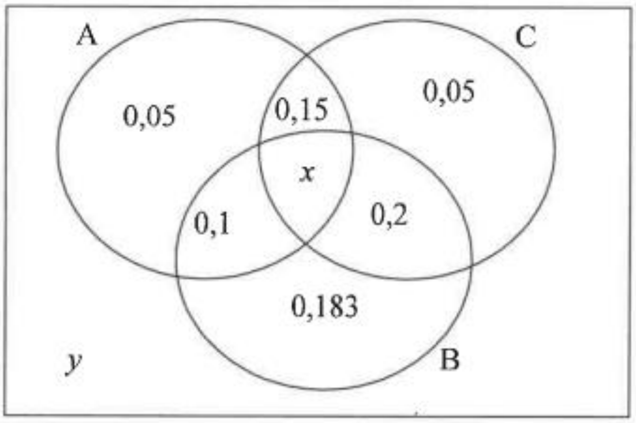
Trousers shirts shoes

This method much quicker!

PROBABILITY

QUESTION 10

10.1 A, B and C are three events. The probabilities of these events (or any combination of them) occurring is given in the Venn-diagram below



10.1.1 If it is given that the probability that at least one of the events will occur is 0,893, calculate the value of:

(a) y, the probability that none of the events will occur. (1)

(b) x, the probability that all three events will occur. (1)

10.1.2 Determine the probability that at least two of the events will take place. (2)

10.1.3 Are events B and C independent? Justify your answer. (5)

10.2 A four-digit code is required to open a combination lock. The code must be even-numbered and may not contain the digits 0 or 1. Digits may not be repeated.

10.2.1 How many possible 4-digit combinations are there to open the lock? (3)

10.2.2 Calculate the probability that you will open the lock at the first attempt if it is given that the code is greater than 5 000 and the third digit is 2. (5)

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QUESTION 10

10.1 Flags from four African countries and three European countries were displayed in a row during the 2021 Olympics.

Determine:

10.1.1 The total number of possible ways in which all 7 flags from these countries could be displayed (2)

10.1.2 The probability that the flags from the African countries were displayed next to each other (3)

10.2 A and B are two independent events.

P(A) = 0,4 and P(A or B) = 0,88

Calculate P(B). (3)

10.3 There are 120 passengers on board an aeroplane. Passengers have a choice between a meat sandwich or a cheese sandwich, but more passengers will choose a meat sandwich. There are only 120 sandwiches available to choose from. The probability that the first passenger chooses a meat sandwich and the second passenger chooses a cheese sandwich is $\frac{18}{85}$. Calculate the probability that the first passenger will choose a cheese sandwich. (5)

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QUESTION 11

After travelling a distance of 20 km from home, a person suddenly remembers that he did not close a tap in his garden. He decides to turn around immediately and return home to close the tap.

The cost of the water, at the rate at which water is flowing out of the tap, is R1,60 per hour.

The cost of petrol is $\left(1{,}2+\frac{x}{4000}\right)$ rands per km, where x is the average speed in km/h.

Calculate the average speed at which the person must travel home to keep his cost as low as possible. **[7]**

QUESTION 12

12.1 A and B are independent events. It is further given that:

P(A and B) = 0,3 and P(only B) = 0,2

12.1.1 Are A and B mutually exclusive? Motivate your answer. (1)

12.1.2 Determine:

(a) P(only A) (4)

(b) P(not A or not B) (2)

12.2 A teacher has 5 different poetry books, 4 different dramas and 3 different novels. She must arrange these 12 books from left to right on a shelf.

12.2.1 Write down the probability that a novel will be the first book placed on the shelf. (1)

12.2.2 Calculate the number of different ways these 12 books can be placed on the shelf if any book can be placed in any position. (2)

12.2.3 Calculate the probability that a poetry book is placed in the first position, the three novels are placed next to each other and a drama is placed in the last position. (4)

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QUESTION 11

11.1 Two events, A and B, are such that:

- Events A and B are independent
- P(not A) = 0,4
- P(B) = 0,3

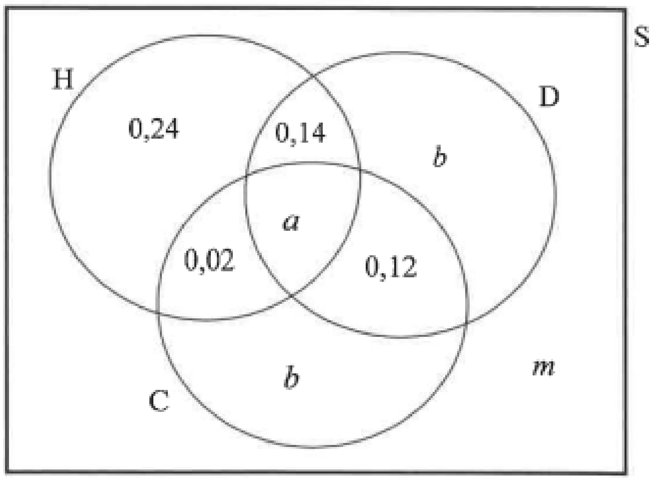
Calculate P(A and B). (3)

11.2 A survey was conducted among 150 learners at a school.

The following observations were made:

- The probability that a learner, selected at random, will take part in:
  - Only hockey (H) is 0,24
  - Hockey and debating (D), but not chess (C) is 0,14
  - Debating and chess, but not hockey is 0,12
  - Hockey and chess, but not debating is 0,02
- The probability that a learner, selected at random, participates in at least one activity is 0,7.
- 15 learners participated in all three activities.
- The number of learners that participate only in debating is the same as the number of learners who participate only in chess.

The Venn diagram below shows some of the above information.



11.2.1 Determine a, the probability that a learner, selected at random, participates in all three activities. (1)

11.2.2 Determine m, the probability that a learner, selected at random, does NOT participate in any of the three activities. (1)

11.2.3 How many learners play only chess? (4)

Worksheet 11 D

3. A car-insurance company is interested in the relationship between the age of drivers and the number of accidents that they have made over a period of 10 years.

   The results of a survey of 1000 drivers is shown in the two-way table below:

Age of driver	Number of accidents		Tot
	5 or less	More than5	
28 years or older	(a)	125	(e)
Younger than 28 years	500	(b)	750
Total	(c)	(d)	(f)

3.1 Calculate the values of (a) to (f). (Not necessarily in this particular order)

3.2 Determine whether the number of accidents made by a person, is independent or not of that person's age. (show calculations to explain your answer)

3.1 $e = 250$ $f = 1000$

$a = 125$ $c = 625$

$b = 250$ $d = 375$

	< 5	> 5	Total
28 years +	125	125	250
< 28 years	500	250	750
Total	625	375	1000

3.2 P(older than 28 years)= $\frac{250}{1000}$

P(less than 5) $= \frac{625}{1000}$

$\frac{250}{1000} \times \frac{625}{1000} = 0{,}15625$

P(older than 28 years and less than 5)

$= \frac{125}{1000} = 0{,}125$

Because P(older than 28 and < 5) $\neq$ P(28+) x P(<5)
the number of accidents made by a person is not independent of the person's age.